Feed-Forward Competitive Shunting Neural Nets: Visualization of Analytical Results

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Abstract

The main objective of this work is to study the characteristics of a biologically plausible neural network model, feed-forward competitive shunting networks. Basic properties of this network is extracted from shunting equations and analytical results are visualized by employing some test cases in experiments.

1 Introduction

One of the most crucial characteristics of human visual perception system is its ability of adaptation in environments which are illuminated in different levels of intensities. Feed-forward competitive shunting neural networks are biologically plausible network models which are used to mimic the behavior of real neuron activations. Shunting networks are models where each inputs are fully connected to the neurons and competitive relations among this neurons determine the activity of each of them. Direct input connections have excitatory effect and lateral connections result in inhibition of the activation of corresponding neurons.

In this study, basic characteristics of such networks will be examined. In the next section, the model and shunting equation will be described in detail. The impact of the equation on the equilibrium state and some problems that may occur in the absence of some terms is described. In Section 3 the basic properties of shunting networks will be examined. The properties are extracted from shunting equation by defining some constraints on parameters. Then, experiments are conducted to test and visualize the analytical results.

2 Feed-Forward Competitive Shunting Networks

Feed-Forward Shunting Networks consist of competitive neurons whose activations are determined by the *membrane equation*. Figure 1 demonstrates a typical network where each input I_i in the input pattern I_i has an excitatory effect on the neuron v_i and inhibitory effect on all other neurons. The activation of any neuron is denoted by x_i and total activation is X. In this paper, I will use some terms from human visual system and real world, directly referring to network values. For example total illumination is used to refer the total input value which is presented to shunting network ($I = \sum_i I_i$). *Pattern* will denote a specific input i in the input pattern and background intensity is the intensity of input pattern without that particular pattern or input i. I will follow the steps in [1] in order to obtain the membrane equation.

The discussion can be initiated from the requirement that system should behave sensitive for high and low intensities. The competitive neurons should be able to adjust their activation for different levels of input pattern values. In order to respond such a need, a maximum activation value B is specified and in each time step, the activation of the neuron v_i is increased at a rate which is proportional to input intensity I_i and the remaining activation value for each cell to a maximum, $B - x_i$. Thus, the rate of change

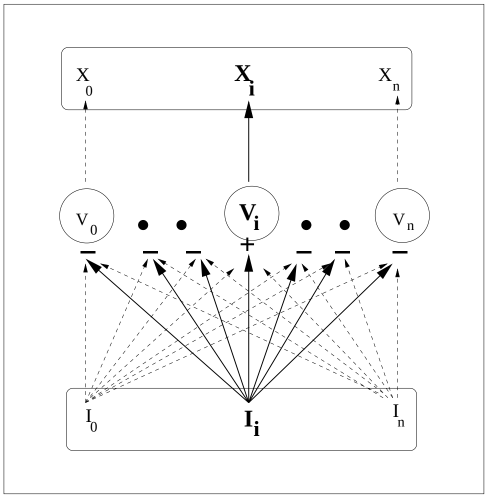

Figure 1: The structure of the Feed-Forward Competitive Neural Network. I_i denotes the i^{th} input value, v_i is the i^{th} neuron and x_i is the activation of i^{th} neuron.'+' sign indicates the excitatory effect of input I_i to the neuron v_i. '-' signs indicate the inhibitory effect of input I_i into other neurons.

in activation is formulated:

$$\frac{d}{dt}x_i = (B - x_i)I_i$$

However, as in the case of light intensity which changes dynamically in our life, the input pattern to the network may fluctuate. Above equation the activation of all neurons will reach to the maximum after some period and neurons cannot respond to input values which are decreased. In order to respond different input values, a decay term is inserted into formula, which continually decrease the activation of each neuron independent from others at a rate which is proportional to the output activation of the units:

$$\frac{d}{dt}x_i = -Ax_i + (B - x_i)I_i$$

In Section 1, the network is proposed to give same response for an input I_i independent of the total intensity if the input ratio to the total intensity remains same. If θ_i denotes the ratio of i^{th} input to the total, $\theta_i = (I_i/I)$, the activation of a neuron in equilibrium can be written as:

$$x_i = \frac{B\theta_i I}{A + \theta_i I}$$

When system reaches equilibrium, the activation of each neuron saturates to a maximum value of B, if total input (background intensity) is continually increased. A competitive mechanism network units is employed in order to avoid from saturation problem. While the i^{th} input I_i excites the neuron v_i, all other inputs $I_{j\neq i}$ will inhibit the activation of the i^{th} unit:

$$\frac{d}{dt}x_i = -Ax_i + (B - x_i)I_i - (x_i + C)\sum_{j\neq i} I_j \qquad (1)$$

Using above equation, unit activation is reformulated when system reaches equilibrium:

$$x_i = \theta_i \frac{BI}{A + I}$$

The activation of each input is independent of the total intensity. Therefore no matter the value of total intensity, x_i is determined only by θ_i, the ratio of I_i/I. The decay rate of activation is called *gain* of x_i. The gain is automatically adjusted according to the inhibitory activations which solves the saturation problem. In ,this argument will be supported by experimental results. C is the hyper-polarization term which specifies the minimum value that an neuron can receive. Therefore, the activation value of each neuron is bounded in the range $[-C, B]$.

As a last point, since I use an iterational approach to enable the convergence of equilibrium states, I should define the Δx_i:

$$\Delta x_i = \frac{d}{dt}x_i \times t$$

where t is time period and will be adjusted for each experiment. It is smaller than 0.001 in all experiment.

3 Experiments

Experiments are mainly conducted in order to test and visualize the theoretical results of the shunting

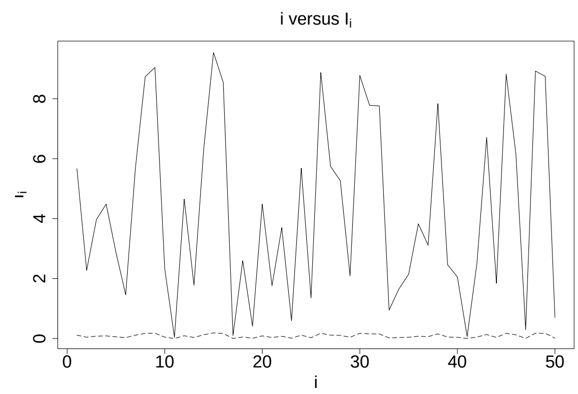

Figure 2: Two different input patterns. This two input patterns differ only by the total input value, the ratio of each input to the total is same for both patterns. Input values varies in the range of $[0, 10]$ and $[0, 0.1]$ for solid and dashed line respectively.

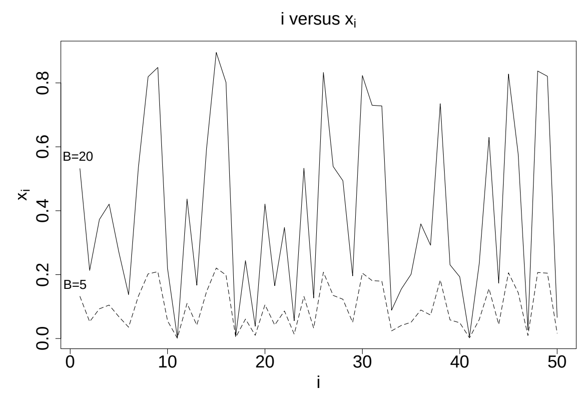

Figure 3: Activation values for each neuron indexed by i. Input patterns in Figure 2 are presented to network, solid line differs from dashed line by the value of parameter B. The equilibrium is obtained after 1000 steps.

network discussions. In this work, I will study the basic and simplest characteristics of feed-forward competitive shunting networks.

One of the main characteristics of this model is the importance of adjusting the parameters in order to obtain various responds for different input patterns. Equation 1 is used to Tuning the parameters and presenting different input patterns results in the response of the network in the means of activation of neurons differ

Automatic gain control and normalization

As two related characteristics of human visual system, *brightness constancy* and *brightness contrast* [1], they both deal with the connection of background illumination, a pattern's real intensity and perceived brightness. This relation is analyzed in various illuminations. Brightness constancy illustrates that the perceived brightness of a pattern is independent of total intensity of the environment and is determined and fixed by the ratio of pattern intensity to background intensity. Brightness contrast rephrase this argument in a different manner, since total perceived brightness is conserved, same patterns which are located in different backgrounds seem to have different intensities. An object looks darker if it is placed on a luminous background.

In order to obtain such a behavior, presented inputs should be normalized into a specified range. For this discussion, parameter C in Equation 1 is assumed to be zero. When the system reaches an equilibrium, if total input is much greater than parameter A, the activation of i^{th} neuron can be computed as,

$$I \gg A \Rightarrow x_i \approx B\frac{I_i}{I}$$

Above equation shows that some constraints on parameters results in automatic gain control mechanism since pattern value is preserved independent of the total input value I, since both terms of above approximation are constant. Next, total is normalized with same parameter constraints, total activation (X) is:

$$I \gg A \Rightarrow X = \sum_i x_i \approx B$$

In the simulations, the random input pattern which is illustrated in Figure 2 is presented into the network. The values of each input of solid plotted pattern is

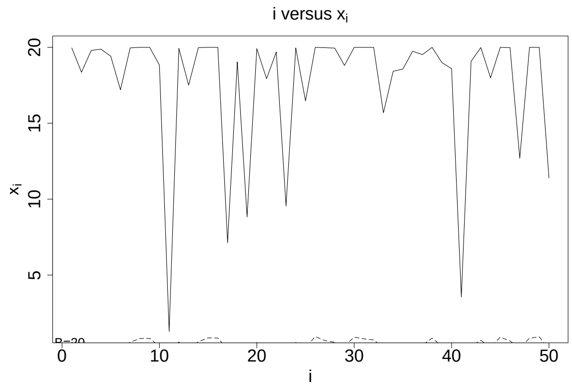

Figure 4: Activation values for each neuron indexed by i. Because adaptive decay term for gain control is not used and Equation 2 is employed, saturation problem arises.

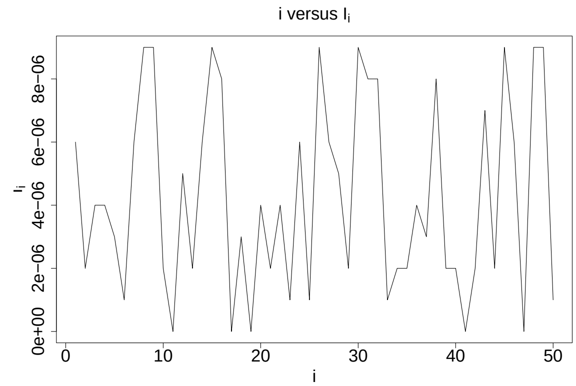

Figure 5: Input pattern which is presented to shunting network for noise suppression experiment.

exactly 100 times the dashed one. Figure 3 demonstrates the activation values x_i for this patterns in equilibrium. First concentrate on solid plot. Both inputs which is demonstrated in Figure 2 are presented to the network independently. Since the ratio of input values to the total is same for both patterns, activation is normalized in equilibrium state according to the parameter B. Thus the activation values of both patterns are matched. The dashed plot in Figure 3 differs from the solid one by the value of B. Since $B = 5$ for solid plot and $B = 20$ for dashed one, the activation values of dashed plot are exactly four time of solid plot. In the lack of automatic gain control mechanism, if Equation 2 is used to update x_i values, saturation problem occurs as shown in Figure 4. x_i values for each neuron become closer and closer to B(20), which was theoretically proved in Section 2.

Noise suppression

While human visual system always performs noise suppression functionality, in image pre-processing steps noise suppression algorithms should be applied in order to obtain high quality images. In this section, first the theoretical background of noise suppression property of membrane equation will be presented and then experiments will be conducted to visualize the effect of the network. C parameter of Equation 1 is assumed to be zero again. If passive decay constant A is selected as much greater than total input, $\sum_i I_i$ (I, activation of i^{th} neuron converges to:

$$A \gg I \Rightarrow x_i = B\frac{I_i}{A}$$

which results in an effect where activation of each neuron, x_i converges some specific values according to the parameter set. Input pattern which is demonstrated in Figure 5 is converged to the activation pattern which is demonstrated in Figure 6. In this experiments, $A = 10$, $B = 10^{-4}$, $t = 10^{-4}$ and range of input values is $[0, 10^{-5}$. If passive decay rate is increased to 20, the activation values can get only one of the two values which is shown in Figure 7.

Suppression of uniform inputs

The other property of shunting networks is its ability to suppress uniform inputs. When the equilibrium is obtained employing the membrane Equation 1 and $\frac{d}{dt}x = 0$, activation of each neuron converges to:

$$x_i = \frac{(B+C)I}{A+I}(\theta_i - \frac{C}{B+C}$$

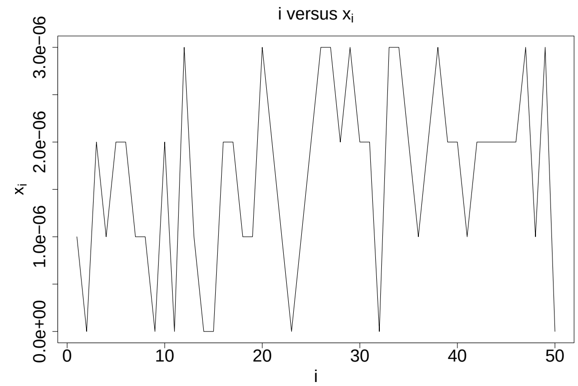

Figure 6: Activation pattern, result of the noise suppression, where A=10.

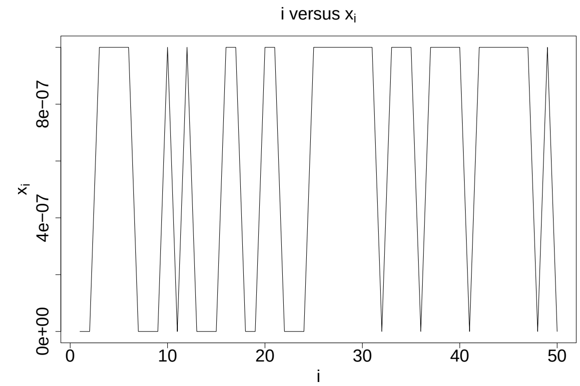

Figure 7: Activation pattern, result of the noise suppression, where A=20.

The constant $\frac{C}{B+C}$ is defined as *adaptation level* in [1]. According to above equation, in order to increase the activation value of any neuron i, adaptation level term should be smaller than I_i/I. For a uniform pattern, if N is the length of the pattern, $\theta_i = 1/N$. If shunting parameters are selected in a manner where adaptation level equals to the ratio $1/N$, for each neuron, activation will converge to zero. In the experiments, the size of the input pattern is 50, thus parameters C and B are set to 1 and 49 (N-1) respectively to obtain such a behavior. Figure 8 demonstrates successive steps where activation of neurons are randomly initialized and a uniform input is applied to the network. At 200^{th} step, all input activation is suppressed and all neurons are shut off. However, a non-uniform input pattern is presented to the shunting network with same parameters, the activation values don't converge to zero (Figure 9).

Shift of dynamic range

Feed-forward competitive shunting networks have the ability of shifting dynamic ranges of their activation values for various illumination levels. In [2], *Grossberg has also shown that the region of maximal sensitivity of a cell shifts without compression as the background intensity is parametrically increased. This is known as the shift property.* The proof of this argument is outside of the scope of this study. The experiments are conducted for different levels of background intensities ($\sum_{j\neq i} I_j$). For each background intensity level, the value of one randomly selected input i is exponentially increased. The activation of i^{th} neuron in response to the increase in i^{th} input is observed. In the experiments, passive decay constant (A) is set to 0.1 and B is initialized to 0.1. C term is again assumed to be zero. 2500 iterations for each intensity level is performed and t is set to 10^{-8}. Simulations are run for background intensities e^{10}, e^{20}, e^{30} and e^{40}. The initial value of i^{th} input is $e^{1.5}$,$e^{3.0}$,$e^{4.5}$ and $e^{6.0}$ respectively. For each intensity level, the I_i is incremented by $e^{0.2}$. Figure 10 demonstrates the shifting property of shunting network.

The maximum value of 0.1 is the value of parameter B. After some period, the i^{th} input has a great dominance over the input pattern, and activation value is preserved in upper saturation point.

4 Conclusion

In conclusion, shunting networks can adaptively respond to total illumination or specific pattern intensity changes over a wide range. They are able to normalize the input pattern, suppress the noise and uniform input. Adjusting the activation range dy-

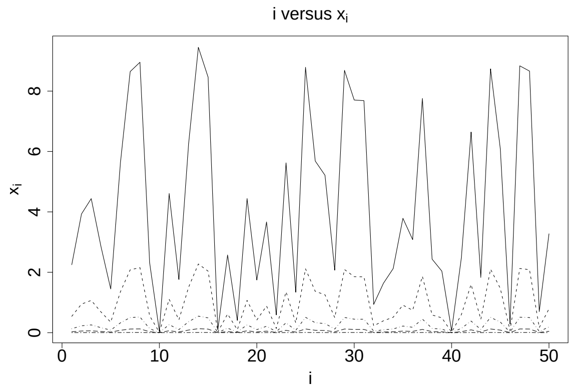

Figure 8: Successive activation patterns, which are obtained in t=0,50,100,150,200 from top to bottom. This figure illustrates the uniform input suppression property of shunting networks.

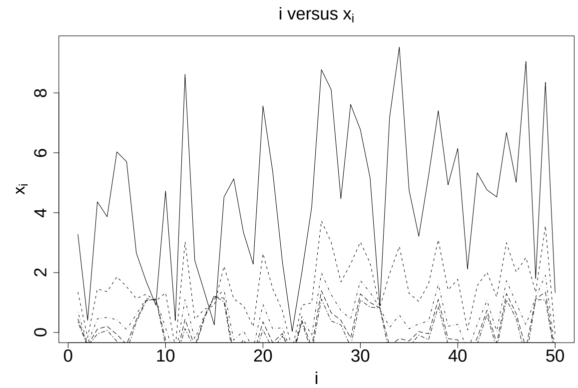

Figure 9: Successive activation patterns, which are obtained in t=0,50,100,150,200 from top to bottom. This figure illustrates that with same parameters that a network can suppress a uniform input, it has no such effect for non-uniform inputs.

namically according to the incoming intensity is one other property which is performed by retinal cells of human eye by the means of shifting the phase of activation when background intensity is modified. This properties of shunting equations are first derived from analytical results then experiments are conducted to test and visualize this results.

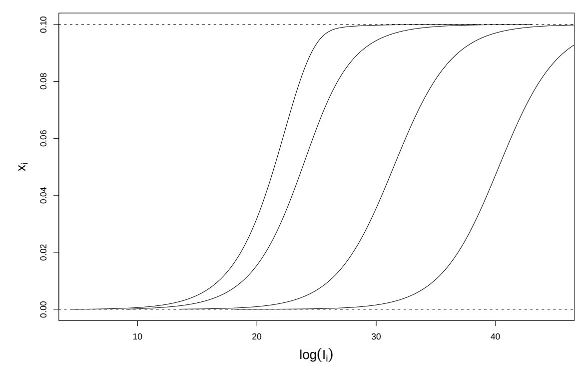

Figure 10: Shift of dynamic range. The shifting curves represent different background intensities. After approximately 1500 steps, i^{th} input value dominates the whole pattern, thus become close to maximum saturation point, B.

References

[1] Stephen Grossberg, 'Why Do Cells Compete? Some Examples From Visual Perception',*Modules and Monographs in Undergraduate Mathematics and its Applications*, UMAP Module 484, 1982.

[2] Paolo Gaudiano, 'Toward a unified theory of spatiotemporary processing in the retina', *Neural Networks and Image Processing*, Ch. 8, 195-220, MIT Press, 1992.